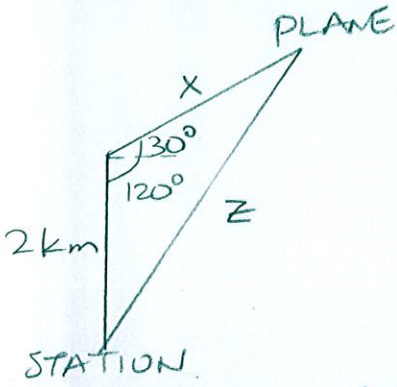
A plane flying with a constant speed of 300 kilometers per hour passes over a ground radar station at an altitude of 2 kilometer and climbs at angle of 30°. At what rate is the distance from the plane to the radar station increasing a minute later ?
You must state the units for the final answer. You do NOT need to show the units during the intermediate steps of your work.

SCORE: ____ / 12 PTS



$\frac{dx}{dt} = 300 \frac{km}{h}$ (1)

WANT $\left.\frac{dz}{dt}\right|_{x=5\,km}$ (1)

IF YOU INDICATED X WAS THE DISTANCE IN THE DIAGRAM

IF YOU INDICATED Z WAS THE DISTANCE FROM THE STATION TO THE PLANE

$z^2 = x^2 + (2km)^2 - 2(2km)x\cos 120^\circ$

$z^2 = x^2 + 2x\,km + 4\,km^2$ (2) *

$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2\frac{dx}{dt}km$ (2)

$2(\sqrt{39}\,km)\frac{dz}{dt} = 2(5\,km)(300\frac{km}{h}) + 2(300\frac{km}{h})km$

(1) (1) (1) (1)

$\frac{dz}{dt} = \frac{1800}{\sqrt{39}}\frac{km}{h} = \frac{1800\sqrt{39}}{39}\frac{km}{h} = \frac{600\sqrt{39}}{13}\frac{km}{h}$

(1) (1)

(*) OTHER SOLUTIONS POSSIBLE DEPENDING ON WHAT EQUATION YOU USED AT THIS STEP — TALK TO ME

Use an appropriate linear approximation to estimate $\cos^{-1} 0.48$.

SCORE: ____ / 6 PTS

$f(x) = \cos^{-1} x$ NEAR $x = \frac{1}{2}$

$L(x) = f(\frac{1}{2}) + f'(\frac{1}{2})(x - \frac{1}{2})$

$= \frac{\pi}{3} - \frac{1}{\sqrt{1-\frac{1}{4}}}(x - \frac{1}{2})$

$= \frac{\pi}{3} - \frac{2}{\sqrt{3}}(x - \frac{1}{2})$

$\cos^{-1} 0.48 \approx \frac{\pi}{3} - \frac{2}{\sqrt{3}}(0.48 - \frac{1}{2})$ (① ① ①)

$= \frac{\pi}{3} - \frac{2}{\sqrt{3}}(-0.02)$

$= \frac{\pi}{3} + \frac{2}{\sqrt{3}}\frac{1}{50} = \frac{\pi}{3} + \frac{1}{25\sqrt{3}}$ (①)

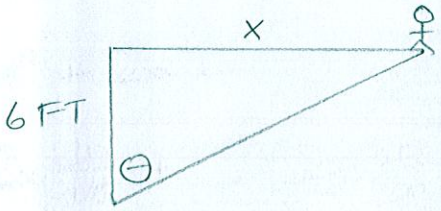
$f'(x) = -\frac{1}{\sqrt{1-x^2}}$ (①)

+① FOR USING $x = \frac{1}{2}$ AS POINT WHERE LINEAR APPROXIMATION IS BASED

A man walks along a straight path at a speed of 5 feet per second. A searchlight is located on the ground 6 feet from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 8 feet from the point on the path closest to the searchlight ?

SCORE: ____ / 12 PTS

You must state the units for the final answer. You do NOT need to show the units during the intermediate steps of your work.



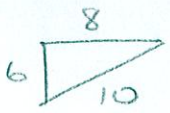
$\frac{dx}{dt} = 5\frac{ft}{sec}$ (①)

WANT $\left.\frac{d\theta}{dt}\right|_{x=8FT}$

① IF YOU INDICATED x WAS THE DISTANCE IN THE DIAGRAM

① IF YOU INDICATED θ WAS THE ANGLE IN THE DIAGRAM

$\tan\theta = \frac{x}{6FT}$ (2) *



$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{6FT}\frac{dx}{dt}$ (2)

(3) $\left(\frac{5}{3}\right)^2 \frac{d\theta}{dt} = \frac{1}{6FT}\frac{5FT}{SEC}$ (①)

$\frac{d\theta}{dt} = \frac{9}{25}\frac{5}{6}$ / SEC

$= \frac{3}{10}$ RADIAN / SEC (① ①)

(*) OTHER SOLUTIONS POSSIBLE DEPENDING ON WHAT EQUATION YOU USED AT THIS STEP - TALK TO ME